Distributed RLGC Transient Model of Coupled Interconnects for Crosstalk Noise Simulation

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Abstract

Noise effects in coupled interconnects, i.e. crosstalk induced glitch and crosstalk induced delay can significantly impact the performance of deep sub-micron (DSM) chips. Therefore, in this paper distributed RLGC transient model of coupled interconnects has been developed that will be useful for analyzing such crosstalk noise effects in DSM chips. The model accuracy is quite comparable to the PSPICE simulation results and yet the simulation speed is at least 11 times faster than the latter.

1. Introduction

Compact expressions for response time and crosstalk of coupled and distributed resistance inductance capacitance (RLC) networks driven by a step input source voltage with arbitrary source resistance are widely referenced in the literature [1]. However, in this paper we derive and simulate the crosstalk coupling noise for mutually coupled, distributed resistance inductance conductance capacitance (RLGC) networks in DSM chips, where all possible sources of coupling noise caused due to mutual capacitance, mutual inductance and also mutual conductance between the neighbouring aggressor-victim lines are taken into account (see Figure-I). This model is usually applicable for very regular IC interconnect structures, and can be used on a wide range of wire lengths and wire dimensions for high-speed semi-global and global interconnects. The advantage of such distributed coupled interconnects' model is that using this single model one can simulate and analyze for various combinations of input signals the crosstalk noise effect on a single victim, which is surrounded by two neighbouring aggressors. Furthermore, because of the consideration of distributed coupled interconnects model the crosstalk model accuracy is very high and yet simulation speed is at least 11 times faster than the PSPICE simulation [2].

The remaining part of the paper is organized as follows: Section -2 describes the coupled interconnects’ model and also derives the victim as well as aggressors’ output voltages in Laplace domain. Thereafter, using the same output voltage expression in Section-3 the (time domain) transient response of the victim line is calculated using various input information. In Section-4 various experimental simulations were performed using Philips CMOS12 (130 nm) technology data and finally, Section-5 presents conclusions.

2. Coupled Interconnects’ (Crosstalk) Model

We describe here the crosstalk fault model of coupled interconnects. The Figure-1 shows the distributed RLGC model of three coupled interconnects, where top and bottom lines are aggressors and middle line is the victim line. It can be seen in the Figure-1 that both top and bottom aggressors are excited, for convenience, with a similar voltage source \( V_{in} \), whereas victim line is excited with a different voltage source \( V_{pt} \).

![Figure-1: Distributed RLGC model of coupled nets.](image)

It can be shown that for such networks, when both aggressors are driven by synchronized rising (or, falling) transitions and the victim line is held constant at static 0 (or, static 1) signal at the driver side the crosstalk positive (or, negative) glitch will be produced due to the existence of strong couplings on the victim line at the receiver side. In contrast, when both aggressors and victim lines are driven by mutually opposite transitions, (e.g., both aggressors are driven by rising transition, whereas victim with falling transition) the crosstalk-delay is produced on the victim line at the far end side signal. The output voltage (in Laplace domain) as derived in the Appendix for the victim and aggressor nets at the receiver side for both cases can be expressed mathematically as:

\[
V_{op}(s) = \frac{3T_1(s)}{(T_1T_4 - T_5T_3)} V_{in}(s) - \frac{3T_2(s)}{(T_1T_4 - T_5T_3)} V_{pt}(s) \quad (1)
\]

\[
V_{ma}(s) = \frac{-3T_1(s)}{(T_1T_4 - T_5T_3)} V_{in}(s) + \frac{3T_2(s)}{(T_1T_4 - T_5T_3)} V_{pt}(s) \quad (2)
\]

The fourth order approximations of numerators are:
\[ T_j(s) = t_{j0} + t_{j1}s + t_{j2}s^2 + t_{j3}s^3 + t_{j4}s^4; j = 1, 2, 3, 4. \]

Obviously, terms \( t_{j0} \) through \( t_{j4} \) are functions of interconnect, driver and receiver parameters and each one is computed by summing one, or more infinite series. The terms \( V_{ip}(s) \) and \( V_{op}(s) \) in (1) and \( V_{id}(s) \) and \( V_{od}(s) \) in (2) represent the input and output signals of victim (passive) and aggressor (active) nets respectively in s-domain. Therefore, \( V_{ai} \) in Figure-1 represents the output voltage of aggressor’s driver (CMOS) for applied input \( V_{ai} \). Similarly, \( V_{oi} \) in Figure-1 represents the output voltage of victim’s driver (CMOS) for applied input \( V_{ip} \).

3. Transient Response Calculation

The transient response of the driver-victim-receiver model can be computed when any two input signals combination \( V_{ai}(s) \) and \( V_{ip}(s) \) is substituted in (1). For instance, in case of rising (0→1) transition at the aggressors’ driver input and static “0” or (0→0) signal in the victim’s driver input substituting \( V_{ai}(s) = V_i \alpha / s(s+\alpha) \), with \( \alpha = 1/\tau \) = reciprocal of time constant, and \( V_i = 1.0 \) or \( V_{ai0} \), and \( V_{ip}(s) = 0 \) in (1) and thereafter, by taking partial fraction and inverse Laplace transformation the Victim’s output voltage at the receiver side is:

\[
V_{op}(t) = \frac{3V_i \alpha}{a_k}(k_{01}e^{-\tau t} + k_{02}e^{-\alpha t} + \ldots k_{0n}e^{-\alpha n}) \tag{3}
\]

where, \( a_k = (t_{1a}t_{3a} - t_{2a}t_{4a}) \) and \( p_1, p_2, \ldots, p_k \) are the system poles. Note that because of static “0” signal at the victim’s input the second term in (1) vanishes and the voltage appearing at the victim’s output is only due to both aggressors’ contribution. Equation (3) represents the crosstalk glitch similar to the one in Figure-2.

4. Crosstalk Noise Simulation

Simulations have been carried out initially using the Philips CMOS12 (130 nm) technology parameters and later also using the parameters reported in [4].

![Fig. 2: Crosstalk glitch simulation](image)

Figure-2 shows that because of the presence of coupling (both capacitive and mutual inductance) the crosstalk positive glitch has been (127 mV) generated on the victim line and the result is quite comparable to the PSPICE simulation. It can be seen from Figure-2 that victim’s output (glitch) is oscillatory in nature and this is mainly due to the presence of strong mutual inductance between the aggressor-victim interconnects, in addition to self inductance of the victim interconnect. Several other simulations were carried out for crosstalk induced delay simulation and the result was found to be also in good agreement with PSPICE (but not reported here due to space restriction).

5. Conclusions

In this paper the crosstalk noise effect on a single victim is derived and analyzed, which is surrounded by two aggressors. All possible sources of coupling noise between the aggressor and victim nets are considered, which include namely capacitive, inductive and conductive. Because of mutual conductance, the same model can be utilized also for simulating resistive bridging fault between the aggressor and victim nets. The simulations results obtained from our crosstalk model were also validated with PSPICE simulation. It has been observed that our crosstalk model provides very accurate result but performs the simulations much faster (at least 11 times) than PSPICE.

Appendix-A: Derivation of victim and aggressors’ outputs

Referring to Figure 1 and considering only an infinitesimally small segment (\( \Delta x \)) of each interconnect we can write the aggressor and victim voltages in Laplace domain respectively as:

\[
\begin{align*}
V_{ai}(s) & = (R + s(L + M_2))I_{ai}(s) + sM_1I_{pi}(s) + V_{a2}(s) \\
V_{ip}(s) & = (R + sL)I_{pi}(s) + 2sM_1I_{ai}(s) + V_{p2}(s) \tag{A.1}
\end{align*}
\]

where, \( R = r/\Delta x \), \( L = l/\Delta x \) and \( r, l \) represent respectively the per-unit-length resistance and self inductance of the interconnect line. The two mutual inductances are \( M_1 = m_1/\Delta x \) and \( M_2 = m_2/\Delta x \) where, \( m_1 \) and \( m_2 \) represent respectively the coupling co-efficient of mutual inductance between aggressor and victim, and the coupling co-efficient of mutual inductance between top and bottom aggressors.

Now, the current equations for aggressor’ and victim will be as follows:

\[
\begin{align*}
I_{ai}(s) & = (G + sC)V_{a2}(s) + I_{a2}(s) \\
I_{pi}(s) & = (G + sC)V_{p2}(s) + I_{p2}(s) \tag{A.2}
\end{align*}
\]

where, \( G = g/\Delta x \), \( C = c/\Delta x \) and \( g, c \) represent respectively the per-unit-length self conductance and self capacitance between the concerned interconnect and substrate ground. Now substituting \( a = (R + sL), b = (G + sC), d = sM_1 \) and \( e = sM_2 \) in the above equations and rearranging them in the matrix equation form we have:
Note that both coupling co-efficients are less than 1, e.g. 0.3 or 0.4. Also, note that for the two (aggressor-victim) interconnects.

Now, from equations (A.3) and (A.4) we can write the following two matrix equations:

\[
\begin{bmatrix}
2V_{a1} + V_{p1} \\
2I_{a1} + I_{p1}
\end{bmatrix} = \begin{bmatrix}
1 + (a + d + e)b \\
b
\end{bmatrix} \begin{bmatrix}
2V_{a2} \\
2I_{a2}
\end{bmatrix} + \begin{bmatrix}
2bd \\
d
\end{bmatrix} \begin{bmatrix}
V_{p2} \\
I_{p2}
\end{bmatrix}
\]

(A.3)

\[
\begin{bmatrix}
V_{a1} - V_{p1} \\
I_{a1} - I_{p1}
\end{bmatrix} = \begin{bmatrix}
1 + (a + e - 2d)b \\
b
\end{bmatrix} \begin{bmatrix}
V_{a2} \\
I_{a2}
\end{bmatrix} - \begin{bmatrix}
0 \\
d
\end{bmatrix} \begin{bmatrix}
V_{p2} \\
I_{p2}
\end{bmatrix}
\]

(A.4)

Now, from equations (A.3) and (A.4) we can write the following two matrix equations:

\[
\begin{bmatrix}
2V_{a1} + V_{p1} \\
2I_{a1} + I_{p1}
\end{bmatrix} = \begin{bmatrix}
1 + (a + d + e)b \\
b
\end{bmatrix} \begin{bmatrix}
2V_{a2} \\
2I_{a2}
\end{bmatrix} + \begin{bmatrix}
2bd \\
d
\end{bmatrix} \begin{bmatrix}
V_{p2} \\
I_{p2}
\end{bmatrix}
\]

(A.5)

\[
\begin{bmatrix}
V_{a1} - V_{p1} \\
I_{a1} - I_{p1}
\end{bmatrix} = \begin{bmatrix}
1 + (a + e - 2d)b \\
b
\end{bmatrix} \begin{bmatrix}
V_{a2} \\
I_{a2}
\end{bmatrix} - \begin{bmatrix}
0 \\
d
\end{bmatrix} \begin{bmatrix}
V_{p2} \\
I_{p2}
\end{bmatrix}
\]

(A.6)

Note that both coupling co-efficients are less than 1, i.e. 0 < (m₁, m₂) < 1 and they can assume some small values e.g. 0.3 or 0.4. Also, note that for m₁ = m₂ will give rise to M₁ = M₂, which implies a = e. In such case (A.5) and (A.6) matrix equations reduce to (A.7) and (A.8) respectively.

\[
\begin{bmatrix}
2V_{a1} + V_{p1} \\
2I_{a1} + I_{p1}
\end{bmatrix} = \begin{bmatrix}
1 + (a + d + e)b \\
b
\end{bmatrix} \begin{bmatrix}
2V_{a2} \\
2I_{a2}
\end{bmatrix} + \begin{bmatrix}
2bd \\
d
\end{bmatrix} \begin{bmatrix}
V_{p2} \\
I_{p2}
\end{bmatrix}
\]

(A.7)

\[
\begin{bmatrix}
V_{a1} - V_{p1} \\
I_{a1} - I_{p1}
\end{bmatrix} = \begin{bmatrix}
1 + (a + e - 2d)b \\
b
\end{bmatrix} \begin{bmatrix}
V_{a2} \\
I_{a2}
\end{bmatrix} - \begin{bmatrix}
0 \\
d
\end{bmatrix} \begin{bmatrix}
V_{p2} \\
I_{p2}
\end{bmatrix}
\]

(A.8)

Referring to Figure-1 once again and considering now the nodes x₁ (or, x₂) and y we can write the corresponding voltages in Laplace domain as:

\[V_{a2} = V_{a2f}, \quad V_{p2} = V_{p2f}\]

Therefore,

\[\left(2V_{a2} + V_{p2}\right) = \left(2V_{a2f} + V_{p2f}\right)\]

\[\left(V_{a2} - V_{p2}\right) = \left(V_{a2f} - V_{p2f}\right)\]

(A.9)

Now considering the currents through mutual conductance and mutual capacitance between the nodes x₁ (or, x₂) and y we can write:

\[I_{a2} = I_{cm} + I_{a2f}, \quad I_{cm} = f\left(V_{a2} - V_{p2}\right)\]

where, \(f = \left(g_m + sc_m\right)\Delta x\), with \(g_m\) and \(c_m\) being the mutual conductance and mutual capacitance respectively between the two (aggressor-victim) interconnects.

Similarly,

\[I_{p2} = -2I_{cm} + I_{p2f}\]

Therefore,

\[\left(2I_{a2} + I_{p2}\right) = \left(2I_{a2f} + I_{p2f}\right)\]

\[\left(I_{a2} - I_{p2}\right) = 3f\left(V_{a2f} - V_{p2f}\right) + I_{a2f} - I_{p2f}\]

(A.10)

Therefore, from (A.9) and (A.10) we can write:

\[
\begin{bmatrix}
2V_{a2} + V_{p2} \\
2I_{a2} + I_{p2}
\end{bmatrix} = \begin{bmatrix}
1 \\
0
\end{bmatrix} \begin{bmatrix}
2V_{a2f} + V_{p2f} \\
2I_{a2f} + I_{p2f}
\end{bmatrix}
\]

(A.11)

Now, applying (A.11) and (A.12) in matrix equations (A.7) and (A.8) respectively we have:

\[
\begin{bmatrix}
2V_{a1} + V_{p1} \\
2I_{a1} + I_{p1}
\end{bmatrix} = \begin{bmatrix}
1 + (a + d + e)b \\
b
\end{bmatrix} \begin{bmatrix}
2V_{a2f} + V_{p2f} \\
2I_{a2f} + I_{p2f}
\end{bmatrix}
\]

(A.13)

\[
\begin{bmatrix}
V_{a1} - V_{p1} \\
I_{a1} - I_{p1}
\end{bmatrix} = \begin{bmatrix}
1 + (a + e - 2d)(b + 3f) \\
b
\end{bmatrix} \begin{bmatrix}
V_{a2f} - V_{p2f} \\
I_{a2f} - I_{p2f}
\end{bmatrix}
\]

(A.14)

Note that matrix equations (A.13) and (A.14) represent the corresponding ABCD model of first \(\Delta x\) segment of coupled interconnects as shown in Figure-1 in combination and differential mode. Now the corresponding ABCD model of entire length of coupled interconnects can be found easily by cascading \(n\) such small segments each of length \(\Delta x = h/n\), where \(h\) = total interconnect’s length. Therefore,

\[
\begin{bmatrix}
2V_{a1} + V_{p1} \\
2I_{a1} + I_{p1}
\end{bmatrix} = \begin{bmatrix}
1 + (a + d + e)b \\
b
\end{bmatrix} \begin{bmatrix}
2V_{a2f} + V_{p2f} \\
2I_{a2f} + I_{p2f}
\end{bmatrix}
\]

(A.15)

\[
\begin{bmatrix}
V_{a1} - V_{p1} \\
I_{a1} - I_{p1}
\end{bmatrix} = \begin{bmatrix}
1 + (a + e - 2d)(b + 3f) \\
b
\end{bmatrix} \begin{bmatrix}
V_{a2f} - V_{p2f} \\
I_{a2f} - I_{p2f}
\end{bmatrix}
\]

(A.16)

Now, the \(n\)th power \((n\rightarrow\infty)\) of both ABCD (square) matrices can be computed easily as shown in [2]. Therefore,

\[
\lim_{n \to \infty} \left[1 + (a + d + e)b \right]^{n} = \frac{\cosh(y, h)}{\sinh(y, h)} Z_c \sinh(y, h) \cosh(y, h)
\]

\[
\lim_{n \to \infty} \left[1 + (a + e - 2d)(b + 3f) \right]^{n} = \frac{\cosh(y, h)}{\sinh(y, h)} Z_c \sinh(y, h) \cosh(y, h)
\]

(A.17)

(A.18)
where, \( Z_a, Z_p \) are respectively the characteristic impedances in (A.17) and (A.18). Similarly, \( \chi \) and \( \gamma \) represent the propagation constants in (A.17) and (A.18) respectively. From (A.15) and (A.16) we have:

\[
2V_{a1} + V_{p1} = A_1(2V_{oa} + V_{op}) + A_2(2I_{oa} + I_{op}) \tag{A.19}
\]

\[
(V_{a1} - V_{p1}) = B_1(V_{oa} - V_{op}) + B_2(I_{oa} - I_{op}) \tag{A.20}
\]

\[
2I_{a1} + I_{p1} = A_1(2V_{oa} + V_{op}) + A_4(2I_{oa} + I_{op}) \tag{A.21}
\]

\[
(I_{a1} - I_{p1}) = B_1(V_{oa} - V_{op}) + B_4(I_{oa} - I_{op}) \tag{A.22}
\]

where,

\[
A_1 = \cosh(\gamma r, h), \quad A_2 = Z_c \sinh(\gamma r, h), \quad A_3 = \frac{1}{Z_c} \sinh(\gamma r, h), \quad A_4 = \cosh(\gamma r, h), \quad B_1 = \cosh(\gamma r, h), \quad B_2 = Z_f \sinh(\gamma r, h), \quad B_3 = \frac{1}{Z_f} \sinh(\gamma r, h), \quad B_4 = \cosh(\gamma r, h).
\]

Also note that \( V_{oa} = V_{a(n+1)y} \) and \( I_{oa} = I_{a(n+1)y} \) represent respectively the voltage across and current into the aggressor (receiver) load \( C_{La} \) (subscripts ‘a’ and ‘p’ refer to aggressor and victim related terms respectively) [2]. Similarly, \( V_{op} = V_{p(n+1)y} \) and \( I_{op} = I_{p(n+1)y} \) represent respectively the voltage across and current into the victim (receiver) load \( C_{Lp} \). Further noting that currents entering into the receivers are \( I_{oa} = sC_{La}V_{oa} \) and \( I_{op} = sC_{Lp}V_{op} \) (see [2]) we can rewrite (A.19) and (A.20) as follows:

\[
2V_{a1} + V_{p1} = 2(A_1 + sA_2 C_{La})V_{oa} + (A_1 + sA_2 C_{Lp})V_{op} \tag{A.23}
\]

\[
V_{a1} - V_{p1} = (B_1 + sB_2 C_{La})V_{oa} - (B_1 + sB_2 C_{Lp})V_{op} \tag{A.24}
\]

Similarly, from (A.21) and (A.22) we have:

\[
2I_{a1} + I_{p1} = 2(A_1 + sA_4 C_{La})V_{oa} + (A_3 + sA_4 C_{Lp})V_{op} \tag{A.25}
\]

\[
I_{a1} - I_{p1} = (B_3 + sB_4 C_{La})V_{oa} - (B_3 + sB_4 C_{Lp})V_{op} \tag{A.26}
\]

Solving the above equations we get,

\[
I_{a1} = PV_{oa} + QV_{op} \tag{A.27}
\]

\[
I_{p1} = MV_{oa} + NV_{op} \tag{A.28}
\]

where,

\[
P = \frac{1}{3}[2(A_1 + sA_2 C_{La}) + (B_1 + sB_2 C_{La})]
\]

\[
Q = \frac{1}{3}[(A_3 + sA_4 C_{Lp}) - (B_3 + sB_4 C_{Lp})]
\]

\[
M = \frac{1}{3}[2(A_3 + sA_4 C_{La}) - 2(B_3 + sB_4 C_{La})]
\]

\[
N = \frac{1}{3}[(A_3 + sA_4 C_{Lp}) + 2(B_3 + sB_4 C_{Lp})]
\]

The CMOS driver is modeled as a voltage source driving a resistance \( R_s \), and an output capacitance \( C_s \), whereas the CMOS receiver is modeled as a load capacitance \( C_i [2, 3] \). Therefore, the ABCD models of the drivers of both aggressors and victim are respectively:

\[
\begin{bmatrix}
V_{oa}(s) \\
I_{oa}(s) \\
V_{op}(s) \\
I_{op}(s)
\end{bmatrix} = \begin{bmatrix}
1 + sR_s C_s & R_s & 0 & 0 \\
0 & 1 & 0 & 0 \\
1 + sR_s C_i & R_s & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
V_{a1}(s) \\
I_{a1}(s) \\
V_{p1}(s) \\
I_{p1}(s)
\end{bmatrix} \tag{A.31}
\]

Equations (A.27) and (A.31) yield

\[
V_{a1} = \frac{V_{oa} - R_s (PV_{oa} + QV_{op})}{1 + sR_s C_s} \tag{A.33}
\]

Applying (A.28) in (A.32) similarly we get

\[
V_{p1} = \frac{V_{op} - R_s (MV_{oa} + NV_{op})}{1 + sR_s C_p} \tag{A.34}
\]

Substituting (A.33) and (A.34) in (A.23) and (A.24) and eliminating the aggressor’s output voltage \( V_{oa}(s) \) from both equations we get

\[
V_{op}(s) = \frac{-3T_1(s)}{(T_4 - T_2 T_3)} V_{oa}(s) + \frac{3T_4(s)}{(T_4 - T_2 T_3)} V_{p1}(s) \tag{A.35}
\]

Again substituting (A.33) and (A.34) in (A.23) and (A.24) and eliminating the victim’s output voltage \( V_{op}(s) \) from both equations we get the aggressors’ output at the receiver side as follows:

\[
V_{oa}(s) = \frac{3T_1(s)}{(T_4 - T_2 T_3)} V_{a1}(s) - \frac{3T_4(s)}{(T_4 - T_2 T_3)} V_{p1}(s) \tag{A.36}
\]

where,

\[
T_1(s) = 3R_s M + (2(A_1 + sA_2 C_{La}) - 2(B_1 + sB_2 C_{La})) + (1 + sR_s C_p)
\]

\[
T_2(s) = 3R_s P + (2(A_1 + sA_4 C_{La}) + (B_1 + sB_4 C_{La})) + (1 + sR_s C_s)
\]

\[
T_3(s) = 3R_s N + (A_3 + sA_4 C_{Lp}) + (B_3 + sB_4 C_{Lp}) + (1 + sR_s C_p)
\]

\[
T_4(s) = 3R_s Q + (A_3 + sA_4 C_{Lp}) + (B_3 + sB_4 C_{Lp}) + (1 + sR_s C_s)
\]

**Appendix-B:** Derivation of numerators of victim and aggressors’ outputs

The terms \( A_1, A_2, A_3 \) and \( A_4 \) shown in the Appendix-A in (A.19) and (A.21) can be written as follows:

\[
A_1 = \cosh(\gamma r, h) = 1 + \frac{(\gamma r, h)^2}{2!} + \frac{(\gamma r, h)^4}{4!} + \frac{(\gamma r, h)^6}{6!} + \cdots
\]

Now, the fourth order approximation of \( A_1 \) or \( A_4 \) can be written as follows:

\[
A_1 = \cosh(\gamma r, h) = a_{01} + a_1 \gamma r + a_2 \gamma^2 r + a_3 \gamma^3 r + a_4 \gamma^4 r
\]

Considering

\[
x = r g, \quad y = r c + g l (1 + m_1 + m_2) \quad \text{and} \quad z = c l (1 + m_1 + m_2)
\]

\[
a_{01} = \cosh(\gamma \sqrt{g})\]

\[
a_{11} = \frac{\gamma h}{2\sqrt{g}}\sinh(\sqrt{g}).
\]
Similarly, considering also the fourth order approximation of \( A_2 \) term, we have:

\[
a_2 = Z \sinh (v_f h) = a_{20} + a_{21} s + a_{22} s^2 + a_{23} s^3 + a_{24} s^4
\]

where,

\[
a_{20} = \frac{Z}{2} \sinh (v_f h),
\]

\[
a_{21} = 2 y h \left[ \frac{\cosh (v_f h) + \sinh (v_f h)}{8 (v_f h)} - \frac{\sinh (v_f h)}{8 (v_f h)} \right],
\]

\[
a_{22} = 2 y h \left[ \frac{\sinh (v_f h) - \cosh (v_f h)}{48 (v_f h)^2} + \frac{\sinh (v_f h)}{16 (v_f h)^2} + \frac{\sinh (v_f h)}{16 (v_f h)^2} \right] + y^2 h^2.
\]

Likewise, the fourth order approximation of \( A_3 \) term is as follows:

\[
A_3 = \frac{1}{Z} \sinh (v_f h) = a_{30} + a_{31} s + a_{32} s^2 + a_{33} s^3 + a_{34} s^4
\]

where,

\[
a_{30} = \frac{Z}{2} \sinh (v_f h),
\]

\[
a_{31} = c y h \left[ \frac{\cosh (v_f h) + \sinh (v_f h)}{8 (v_f h)} - \frac{\sinh (v_f h)}{8 (v_f h)} \right],
\]

\[
a_{32} = 4 (cy + gz) h^2 \left[ \frac{\cosh (v_f h) + \sinh (v_f h)}{8 (v_f h)^2} + \frac{\sinh (v_f h)}{16 (v_f h)^2} \right] + 6 g y h^3.
\]

Similarly, considering also the fourth order approximation of \( A_2 \) term, we have:

\[
A_2 = Z \sinh (v_f h) = a_{20} + a_{21} s + a_{22} s^2 + a_{23} s^3 + a_{24} s^4
\]

where,

\[
A_3 = \frac{1}{Z} \sinh (v_f h) = a_{30} + a_{31} s + a_{32} s^2 + a_{33} s^3 + a_{34} s^4
\]

Following the same procedure as above the terms \( B_j \), \( B_j \), \( B_3 \) and \( B_4 \) (i.e. \( b_{10}, b_{20}, \ldots, b_{43}, b_{44} \) co-efficients) shown in Appendix-A in (A.20) through (A.22) can be determined.

The fourth order approximation of \( A_4 \) term is same as \( A_1 \) term, since,

\[
A_4 = \cos (h_f) = a_{40} + a_{41} s + a_{42} s^2 + a_{43} s^3 + a_{44} s^4
\]

Therefore,

\[
a_{40} = a_{10}, \quad a_{41} = a_{11}, \quad a_{42} = a_{12}, \quad a_{43} = a_{13} \quad \text{and} \quad a_{44} = a_{14}
\]

Following the same procedure as above the terms \( B_j \), \( B_j \), \( B_3 \) and \( B_4 \) (i.e. \( b_{10}, b_{20}, \ldots, b_{43}, b_{44} \) co-efficients) shown in Appendix-A in (A.20) through (A.22) can be determined.

The \( P, Q, M \) and \( N \) terms shown in (A.27) and (A.28) of Appendix-A can be computed as follows:

\[
P = \frac{1}{3} \left( (2A_1 + B_1) + sC_{12}(2A_2 + B_2) \right) = p_0 + p_1 s + p_2 s^2 + \ldots + p_4 s^4.
\]
Similarly, for $j = 2$:

$$p_3 = \frac{1}{3}((a_{12} + b_{12}) + C_{la}(a_{14} + b_{14})),$$

where,

$$q_3 = \frac{2}{3}(a_{12} - b_{12} + C_{fp}(a_{14} - b_{14})),$$

$$q_7 = \frac{1}{3}(a_{13} - b_{13} - C_{la}(a_{22} - b_{22})),$$

$$q_4 = \frac{1}{3}(a_{14} - b_{14} + C_{fp}(a_{14} - b_{14})),$$

$$t_1 = 3R_{sp}a_1 + 2(a_{12} - b_{12}),$$

$$t_2 = 3R_{sp}a_2 + 2(a_{13} - b_{13}) - C_{la}(a_{22} - b_{22}),$$

$$t_3 = 3R_{sp}a_3 + 2(a_{14} - b_{14}) - C_{la}(a_{24} - b_{24}),$$

$$t_4 = 3R_{sp}a_4 + 2(a_{14} - b_{14}) - C_{la}(a_{24} - b_{24}).$$

Likewise, for $j = 3$:

$$t_5 = 3R_{sp}a_4 + 2(a_{14} - b_{14}) + C_{la}(a_{24} - b_{24}) + (a_{24} + b_{24})R_C, \quad t_6 = 3R_{sp}a_5 + (a_{11} + b_{11}) + C_{la}(a_{21} + b_{21}) + (a_{21} + b_{21})R_C,$$

$$t_7 = 3R_{sp}a_6 + (a_{11} + b_{11}) + C_{la}(a_{21} + b_{21}) + (a_{21} + b_{21})R_C,$$

$$t_8 = 3R_{sp}a_7 + (a_{12} + b_{12}) + C_{la}(a_{22} + b_{22}) + (a_{22} + b_{22})R_C,$$

$$t_9 = 3R_{sp}a_8 + (a_{12} + b_{12}) + C_{la}(a_{22} + b_{22}) + (a_{22} + b_{22})R_C,$$

Finally, for $j = 4$:

$$t_5 = 3R_{sp}a_4 + (a_{11} - b_{11}) + C_{la}(a_{21} - b_{21}) + (a_{21} - b_{21})R_C,$$

$$t_6 = 3R_{sp}a_5 + (a_{11} - b_{11}) + C_{la}(a_{21} - b_{21}) + (a_{21} - b_{21})R_C,$$

$$t_7 = 3R_{sp}a_6 + (a_{12} - b_{12}) + C_{la}(a_{22} - b_{22}) + (a_{22} - b_{22})R_C,$$

$$t_8 = 3R_{sp}a_7 + (a_{12} - b_{12}) + C_{la}(a_{22} - b_{22}) + (a_{22} - b_{22})R_C,$$

$$t_9 = 3R_{sp}a_8 + (a_{12} - b_{12}) + C_{la}(a_{22} - b_{22}) + (a_{22} - b_{22})R_C,$$

$$T(s) = (T_1T_4 - T_2T_3) = \left(t_1 + t_2 + s + t_2s^2 + \cdots + t_2s^8\right).$$

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References


